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Research Article

Big Bang as a Critical Point

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This article addresses the issue of possible gravitational phase transitions in the early universe. We suggest that a second-order phase transition observed in the Causal Dynamical Triangulations approach to quantum gravity may have a cosmological relevance. The phase transition interpolates between a nongeometric crumpled phase of gravity and an extended phase with classical properties. Transition of this kind has been postulated earlier in the context of geometrogenesis in the Quantum Graphity approach to quantum gravity. We show that critical behavior may also be associated with a signature change in Loop Quantum Cosmology, which occurs as a result of quantum deformation of the hypersurface deformation algebra. In the considered cases, classical space-time originates at the critical point associated with a second-order phase transition. Relation between the gravitational phase transitions and the corresponding change of symmetry is underlined.

1. Introduction

Accumulating results of theoretical investigations indicate that the gravitational field exists in different phases. First indications supporting such an idea came from considerations of three-dimensional Euclidean quantum gravity [1]. By means of Monte Carlo simulations it was possible to explore the configuration of the gravitational field under various conditions. For four-dimensional Euclidean gravity, gravity exhibits two phases: the *crumpled* phase and the *branched polymer* phase [2]. This result has since been generalized to the case of four-dimensional gravity with an imposed causality condition, formulation known as Causal Dynamical Triangulations (CDT). The causality condition turned out to be essential for the correct phase structure of gravity, leading to emergence of the four-dimensional space-time [3]. Generation of such extended phase in the Euclidean approach introducing a nontrivial path integral measure remains an interesting possibility [4]. Furthermore, similarly to a phase structure of the Lifshitz scalar [5], the critical surface of CDT has been divided into three regions separated by the first- and second-order transition lines [6]. Interestingly, a theory describing gravity at a triple point (Lifshitz point) of the phase diagram has been constructed and shown to be power-counting renormalizable [7]. Further evidence for the nontrivial phase structure of gravity comes from Quantum Graphity [8]. This approach utilizes the idea of *geometrogenesis:* a transition between geometric and nongeometric phases of gravity.

The basic question one can ask, assuming the existence of the different phases of gravity, is where can the other phases be found? A natural place to search for them is high curvature regions such as interiors of the black holes and the early universe. Because of a horizon, a possibility of relating phase change inside of black holes with astronomical observations is a difficult task. Nevertheless, gravitational phase transitions occurring under the black hole horizon, including the signature change transition, have been a subject of theoretical studies (see, e.g., [9, 10]). Perhaps empirically more promising is a search for signatures of the gravitational phase transitions which took place in the early universe. We will focus on this direction here.

So far, there has been very little attention devoted to this issue in the literature. Most studies of the phase transitions in the early universe were dedicated to the matter sector, rather than gravity [11, 12]. Among the few studies on the gravitational phase transitions in the early universe, the work of [13, 14] is especially noteworthy. In [13] a specific model of geometrogenesis, through a second-order phase transition, has been proposed. It was shown that, by assuming the holographic principle to be fulfilled in the high temperature phase, it is possible to generate a power spectrum of primordial perturbations that is in agreement with observations. In [14] the cosmological relevance of second-order phase transitions is discussed. Arguments supporting generation of “inflationary” power spectrum from critical behavior of the gravitational field have been presented.

In what follows we attract attention to the fact that a second-order gravitational phase transition has recently been observed within Causal Dynamical Triangulations [15]. The transition takes place exactly between the phases of the form discussed in [8, 13]. Therefore, CDT gives a concrete realization of the scenario of geometrogenesis. We also show that gravitational phase transition may be associated with the deformation of general covariance, recently observed in the context of Loop Quantum Cosmology (LQC). In both cases, the phase transition is of second order, suggesting a critical nature of the emergence of classical space-time in the early universe.

2. Causal Dynamical Triangulations

Analysis performed within four-dimensional CDT with a positive cosmological constant indicates the presence of three different phases of the gravitational field, called A, B, and C [6]. The phases are separated by the first- (A-C) and second-order (B-C) transition lines presumably intersecting at the triple point. The order of the A-B phase transition has not been determined so far.

At large scales, phase C forms the four-dimensional de Sitter space [16]. The phase is, however, not fully classical since it exhibits dimensional reduction to two dimensions at short scales [17]. This can be shown by investigating properties of the spectral dimension, defined via a diffusion process. Nevertheless, the phase C can be associated with the “usual” phase of gravity. The two remaining phases are fundamentally different from this phase. Phase A is characterized by a vanishing interaction between adjacent time slices. Phase B, resembling the *crumpled* phase in Euclidean gravity, is characterized by a large (tending to infinity in the $\infty-$ volume limit) Hausdorff and spectral dimension. Phase B shares features of the high temperature phase postulated in Quantum Graphity. Moreover, this phase is separated with the low energetic phase C by the second-order phase transition. This is in one-to-one correspondence to the Quantum Graphity case. Based on this observation, we hypothesize the following.

Hypothesis 1. In the early universe, there was a second- (or higher) order phase transition from the high temperature nongeometric phase to the low temperature geometric phase of gravity. The transition is associated with a change of the connectivity structure between the elementary chunks of space.

The change of connectivity can be inferred from the considerations of the spectral dimensions of the phases B and C. In order to see it explicitly let us consider a toy model of the universe composed of the N chunks of space. They will be represented by the nodes of a graph. A structure of adjacency is represented by the links.

In phase C, which is a geometric phase, the degree of vertices is low. In our toy model it equals 2 and the resulting space is represented by the Ring graph (see Figure 1(a)). The spectral dimension of this graph can be found by determining spectrum (eigenvalues λ_n) of the Laplace operator $\Delta \equiv A-D$, where A is an adjacency matrix and D is a degree matrix. By using the expression for the trace of the heat kernel K one can find that

$$d_S \equiv -2\frac{\partial \log \mathrm{tr}K}{\partial \log \sigma} = -2\sigma \frac{\sum_{i=1}^{N} \lambda_n e^{\lambda_n \sigma}}{\sum_{i=1}^{N} e^{\lambda_n \sigma}}, \tag{1}$$

where σ is a diffusion time.

In Figure 2 we plot function (1) for the Ring graph, for which the eigenvalues are $\lambda_n = 2(\cos(2\pi n/N) - 1)$. At intermediate diffusion times the spectral dimension is equal to one, as expected classically. The short time behavior, corresponding to dimensional reduction observed in the four-dimensional case, is due to the discrete nature of the network. Furthermore, at large diffusion times the spectral dimension is again falling to zero due to the compactness of space (not visible in Figure 2).

Let us now model the high temperature nongeometric phase B by assuming, for computational simplicity, a maximal degree of nodes. The resulting Complete graph is shown in Figure 1. The assumption of completeness allows one to determine spectrum of the Laplace operator analytically and enables the simplification of (1) to

$$d_S = \frac{2N\sigma (N-1)}{e^{N\sigma} + (N-1)}. \tag{2}$$

We plot this function in Figure 2, comparing it with the low temperature case. As we see, the spectral dimension is now peaked at small diffusion times. The maximal value of the spectral dimension grows with the number of nodes as $d_{S,\max} \approx 2W(N/e)$, where $W(x)$ is the Lambert function. This behavior is in qualitative agreement with the numerical computations performed in four-dimensional CDT [6].

3. Loop Quantum Cosmology

Recent developments in LQC indicate that the hypersurface deformation algebra (HDA) is deformed due to the quantum gravitational effects [18]. This means that the general covariance is quantum deformed, but not broken.

In condensed matter physics, a change of symmetry is often associated with the occurrence of a phase transition. By extrapolating this observation to the sector of gravitational interactions, we make the following hypothesis.

Hypothesis 2. Phases of gravity are distinguished by different types of hypersurface deformation algebras.

In order to support this hypothesis we present an example based on holonomy corrections in LQC. In this case, HDA

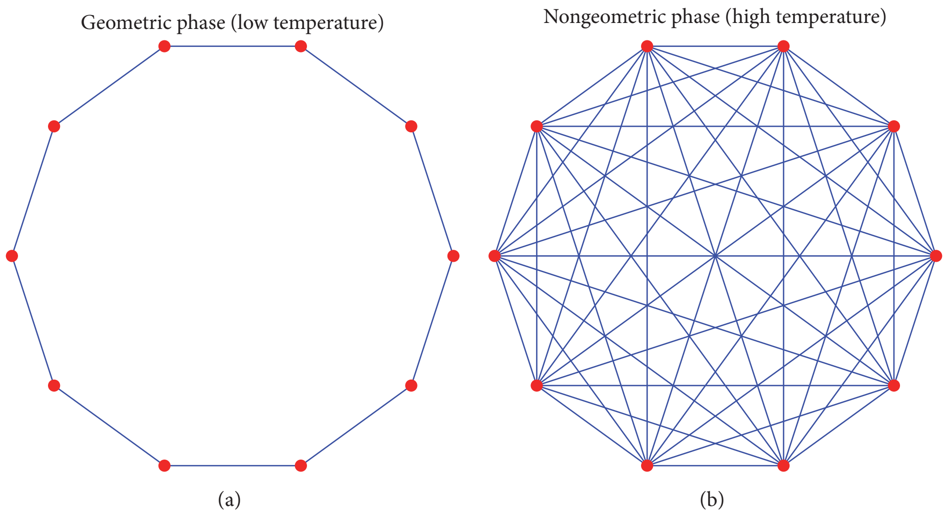

Figure 1: (a) Ring graph being a toy model of the low temperature geometric state of gravity. (b) Complete graph being a model of high temperature nongeometric state of gravity.

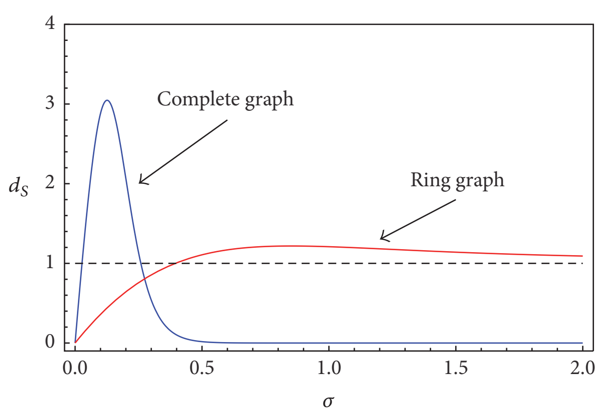

Figure 2: Spectral dimensions for the Ring graph (geometric phase) and the Complete graph (nongeometric phase) with $N = 20$ nodes.

is deformed such that $\{S, S\} = \Omega D$, where S are scalar constraints and D is a diffeomorphism constraint, and the remaining brackets are unchanged [19]. The $\Omega = 1-2(\rho/\rho_c)$ is a deformation factor, ρ denotes energy density of matter, and ρ_c is a maximal energy density expected to be of the order of the Planck energy density.

At low energy densities, the classical Lorentzian HDA with $\Omega = 1$ is recovered, while at $\rho = \rho_c$, $\Omega = -1$ corresponding to Euclidean space. Approaching the Planck epoch is therefore associated with the signature change from Lorentzian space-time to Euclidean four-dimensional space [20]. Interestingly, at $\rho = \rho_c/2$ the HDA reduces to the ultralocal form ($\{S, S\} = 0$) describing a state of *silence* [21]. This state shares properties of phase B in CDT, giving a first indication for the relationship between the phase of gravity and deformation of HDA. This is, in particular, because the ultralocal limit is characterized by collapse of light-cones ($c \to 0$). In CDT, the length of time links becomes dominant over the length of the spatial links while approaching the B phase, which can be interpreted as an effective decrease of the speed of light. Furthermore, in the B phase the space points collapse into the crumpled configuration with no spatial extension. Such behavior is in agreement with properties of the ultralocal limit. Therefore, one can presume that the geometrogenesis related to the second-order phase transition from phase B to phase C is associated with the change of the HDA from the ultralocal one ($\{S, S\} = 0$) to the one characterizing classical gravity ($\{S, S\} = D$). The phase change is, therefore, reflected by an appropriate change of symmetry, which is embedded in the form of HDA. Surprisingly, the deformation of HDA observed in LQC (i.e., $\{S, S\} = \Omega D$) continuously interpolates between the ultralocal and geometric phases.

Worth stressing is that besides the classical limits with either $\Omega = 1$ (Lorentzian manifold) or $\Omega = -1$ (Euclidean manifold), the geometry associated with the deformed HDA is poorly understood. It is known that the deformed algebra leads to gauge transformations which do not commute with the Lie derivatives. This indicates that the standard notion of an invariant metric has to be generalized appropriately and the geometry has non-Riemannian character [22]. This concerns both the deformations observed in the cosmological sector (LQC) as well in the spherical symmetric case. Nevertheless, even if the standard metric loses its meaning in the intermediate regime, Hamiltonian dynamics remains well defined and the structure of causality can be inferred based on analysis of equations of motions and the corresponding characteristic equations (see [23]). This, in particular, concerns the ultralocal state which interpolates between the regions of negative and positive values of the deformation factor Ω.

Further evidence supporting Hypothesis 2 comes from a simple model of the signature change as a spontaneous symmetry breaking (SBB) associated with a second-order phase transition [23]. The symmetry breaking is from $SO(4)$ to $SO(3)$ at the level of an effective homogeneous vector

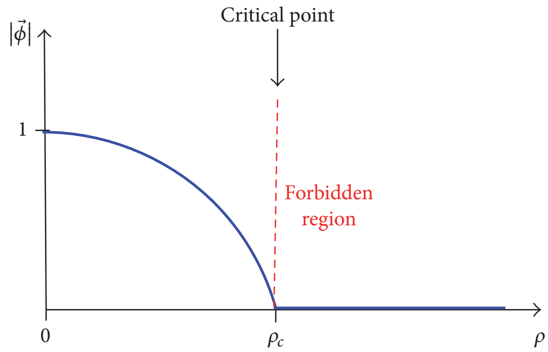

FIGURE 3: Modulus of the filed ϕ^μ as a function of the energy density ρ. The region $\rho > \rho_c$ is forbidden within the model.

field ϕ_μ. This translates to a symmetry change from $SO(4)$ to $SO(3,1)$, experienced by the field living on a geometry described by the metric $g_{\mu\nu} = \delta_{\mu\nu} - 2\phi_\mu\phi_\nu$.

Let us assume that the free energy for the model with a massless scalar field ν is

$$F = \int dV \underbrace{\left(\delta^{\mu\nu} + \frac{2\phi^\mu\phi^\nu}{1 - 2\left|\vec{\phi}\right|^2}\right)}_{g^{\mu\nu}} \partial_\mu \nu \partial_\nu \nu + \beta \underbrace{\left[\left(\frac{\rho}{\rho_c} - 1\right)\left|\vec{\phi}\right|^2 + \frac{1}{2}\left|\vec{\phi}\right|^4\right]}_{V(\phi^\mu,\rho)}, \tag{3}$$

where $|\vec{\phi}| = \sqrt{\delta^{\mu\nu}\phi_\mu\phi_\nu}$ and β is a constant. Due to the symmetry breaking kinetic factor, the expression for the free energy is not explicitly $SO(4)$ invariant. Treating this term as a perturbation (which is valid for sufficiently small values of ν), the equilibrium is obtained by minimizing value of the potential $V(\phi^\mu, \rho)$.

At energy densities $\rho > \rho_c$ the vacuum state maintains the $SO(4)$ symmetry, leading to $|\vec{\phi}| = 0$. The metric elements are therefore $g_{00} = 1$ and $g_{ii} = 1$, representing the four-dimensional Euclidean space. This region is, however, forbidden due to the constraint $\rho \leq \rho_c$ present in LQC. In turn, below the critical energy $\rho \leq \rho_c$, the minimum of the potential is located at $|\vec{\phi}| = \sqrt{1 - \rho/\rho_c}$ in some spontaneously chosen direction (see Figure 3). Without loss of generality, let us assume that the SSB takes place in direction ϕ_0, for which

$$\begin{aligned} g_{00} &= 1 - 2\phi_0\phi_0 = 1 - 2\left(1 - \frac{\rho}{\rho_c}\right) = -1 + 2\frac{\rho}{\rho_c} = -\Omega, \\ g_{ii} &= 1, \end{aligned} \tag{4}$$

leading to the effective speed of light $c_{\text{eff}}^2 = \Omega$. As a consequence, the equation of motion for the scalar field ν takes the form

$$g^{\mu\nu}\partial_\mu\partial_\nu \nu = -\frac{1}{c_{\text{eff}}^2}\frac{\partial^2}{\partial t^2}\nu + \Delta\nu = 0, \tag{5}$$

manifesting $SO(4)$ symmetry at the critical point ($\Omega = -1$) and $SO(3,1)$ symmetry in the low temperature limit ($\Omega = 1$). The form of (5) agrees with the one derived from the holonomy deformations of the HDA [18].

In LQC, energy densities above ρ_c cannot be reached. Therefore, the evolution starts at the critical point located at ρ_c. An interesting possibility is that the system has been maintained at the critical point before the energy density started to drop. This may not require a fine-tuning if the dynamics of the system exhibited Self-Organized Criticality (SOC) [24], which is observed in various complex systems. Interestingly, this concept has already been applied to quantum gravity, however, in order to describe classical configuration of space [25]. Furthermore, it is possible that the region $\rho > \rho_c$ is described by a nongeometric phase of gravity, whose properties cannot be captured within the presented model.

4. Conclusions

We have shown that CDT offers a concrete realization of geometrogenesis, having a second-order gravitational phase transition between the nongeometric and geometric phase. We have explained how measurements of the spectral dimension are related in this case with connectivity of the chunks of space. The critical nature of the emergence of classical space-time in the early universe may give a first possibility of testing CDT. However, this would require more detailed investigations of the properties of the second-order phase transition in CDT. In LQC, which is an alternative to CDT, a gravitational second-order phase transition may explain a signature change in the Planck epoch. In the presented toy model, the universe originates just at the critical point. Furthermore, we have indicated that the phase transition from the nongeometric phase B in CDT to the phase C can be reflected by the deformation of the hypersurface deformation algebra observed in LQC. Interestingly, the latest results in CDT indicate the existence of a new subphase of the phase C [26], characterized by a "bifurcation" of the kinetic term. This behavior resembles the signature change observed in LQC.

Conflicts of Interest

The author declares that they have no conflicts of interest.

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